AP Calculus AB
Test: The Integral and Fundamental Theorem of Calculus
Instructor's Version

Free-Response Question 1 — no calculator (15 points)

1. Measurements of a lake's width were taken at 15-foot intervals, as shown:

f	0	15	30	45	60	75	90	105	120
$f(x)$	0	15	18	20	19	23	24	22	12

Estimate $\int_0^{120} f(x)dx$ with n = 4, using each of the following methods:

A. Left-hand approximation

Answer:

LRAM= $30[0+18+19+24]=1830$

1.25 points for multiplying by width of interval (30)
1 point for using left endpoints only
1.5 points for answer

B. Right-hand approximation

Answer:

RRAM= $30[18+19+24+12]=2190$

1.25 points for multiplying by width of interval (30)
1 point for using left endpoints only
1.5 points for answer

C. Trapezoidal approximation

Answer:

TRAP= $15[0+2(18)+2(19)+2(24)+12]=2010$

1.25 points for multiplying by half the height (15)
1 point for using trapezoid formula
1.5 points for answer

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D. Midpoint approximation

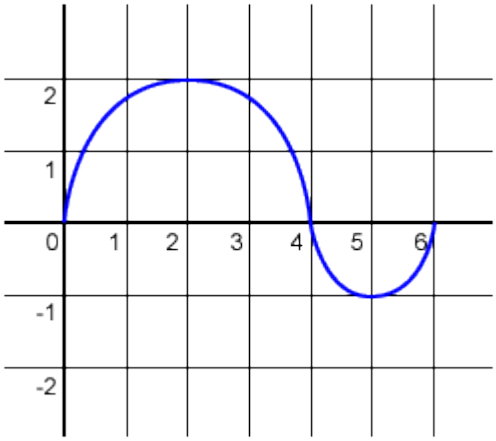
Answer:

MRAM= $30[15+20+23+22]=2400$

1.25 points for multiplying by width of interval (30)
1 point for using midpoints only
1.5 points for answer

Free-Response Question 2 — with calculator (15 points)

2. Let f be the function shown below, with domain the closed interval [0, 6]. The graph of f is shown below, composed of two semicircles. Let $h(x)=\int_0^{2x-1} f(t)dt$



A. Determine the domain of $h(x)$.

Answer:

$\left\lfloor \frac{1}{2}, \frac{7}{2} \right\rfloor$ (1 point).

Justification: $2x-1=0$ (2 points) $\Rightarrow x=\frac{1}{2}$

$2x-1=6$ (2 points) $\Rightarrow x=\frac{7}{2}$

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B. Find $h'\left(\frac{5}{2}\right)$.

Answer:

0 (1 point).

Justification:

$h'(x)=\frac{d}{dx}\int_0^{2x-1} f(t)dt = 2f(2x-1)$ (2 points)

$h'\left(\frac{5}{2}\right)=2f\left(2\left(\frac{5}{2}\right)-1\right)=2f(4)=2(0)$ (2 points)

C. At what x is $h(x)$ a maximum? Show all the analysis that leads to your conclusion.

Answer:

$x=\frac{5}{2}$ (1 point)

Explanation: Critical points for $h(x)$ are where $h'(x)=\frac{d}{dx}\int_0^{2x-1} f(t)dt = 2f(2x-1)=0$ or is undefined. The critical points occur when $2x-1=4$ or $x=\frac{5}{2}$, or at the endpoints, $x=\frac{1}{2}$ and $x=\frac{7}{2}$. (1 point)

x	$\left(\frac{1}{2},\frac{5}{2}\right)$	$\frac{5}{2}$	$\left(\frac{5}{2},\frac{7}{2}\right)$
$h'(x)$	Positive	0	Negative
$h(x)$	Increasing	Max	Decreasing

Note: 3 points for the analysis of the derivative and the function.

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